SCAFFOLDING AND INVESTIGATION OF THINKING TO CALCULUS PROJECT IN PROJECT BASED LEARNING (PBL) ACTIVITIES

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ABSTRACT

PBL adoption is increasingly widespread and recognized for various disciplines in higher education and in basic education has made a trend in learning innovation today. PBL provides individuals with freedom to find, utilize, build knowledge through experience in the form of project completion. The characteristics of the project used are the existence of investigations that give students the freedom to change and build knowledge. The investigation took place every day in a row from the student team (2 people) for 4 days for the given calculus project. The results of project completion investigations were delivered through team presentations, discussions, interviews with lecturers, and the provision of scaffolding by lecturers. Investigation which success include combination, definition of maclaurin series, tribe and series and derivative function which support project finishing, describing material supporting of project through example, verification binomial theorem, identification to 30 tribes maclaurin series/binomial series and approach value through tool of conting/software (excel, calculator, geogebra). Scaffolding thinks incompatibility occurs in understanding and interpretation of combination notation, thought theorem, and implementation in calculations. Researcher's discovery that students are very active in building knowledge and exploring their experiences, as well as building openness in teams and instructors.

1. Introduction

PBL in this study as Project Based Learning. We can see PBL in other research [1]–[3]. In addition, the term PBL is also used as Problem Based Learning [4]–[6]. In 1969 Problem Based Learning began at McMaster University in Canada for the study of medicine initiated by Don Woods in his research [5], [7]. Coming a tradition of project pedagogy in engineering educational emerged on Denmark which include learning concept by doing and learning is based experience to be first introduction of PBL [5].

Learning is no longer seen as a transfer process but rather to the result of construction by students through social interaction, with their environment, and learning resources. Education concepts like 'discovery learning' learning by doing ', 'experiential learning ', and' student center learning 'clearly suggest exploiting human traits like curiosity and the sense of mastery and self-determination (Rogers, 1961; Kolb, 1984; Schmidt, 1983 available at [5]). Both PBL and Problem Based learning are inquiry based learning, which is based on constructivism, allowing students active exploration and building knowledge challenges experience [4]”. PBL is an extension of problem based learning [3].

The beginning of PBL was adopted more slowly in higher education but in its development it was suggested that learning adopt PBL [8]. The results of research on PBL have had an increasingly recognized impact and have extended to various fields of disciplines in higher education [4]. Higher education that adopted PBL included medical education (Stinson & Milter), chemical engineering (Woods),
architecture (Kingsland), economics (Gijselaers), education (Hmelo-Silver) pre-service teachers, Mathematics and science [7, 9]. Other adoptions are for elementary schools, middle schools, high schools, universities, and professional schools.” [6, 7, 9] and experiential education [10].

In PBL, the project has characteristics, namely (1) the project work group is smaller than problem based learning, (2) the project task is closer to reality and takes longer, (3) project assignments produce products [5]. The beginning of the project is defined as complex units and tasks that require more resources than can be given by one person (Green-Ussing, 1990 quoted in [5]). Some examples of projects that match PBL include Riser Laptops [5]. In its development, forms of investigation involving transformation and knowledge construction are examples of projects in PBL [11]. During the process of completing project work by students, the lecturer becomes an advisor, consultant [2] and facilitator [1].

Students who complete the project are involved in the team. Through this collaboration, it has the potential to provide opportunities to carry out discussions, communicate, and learn collaboratively, exchange ideas and experiences that they already have. By completing the project students get a very valuable and long lasting experience. That experience includes the ability to investigate, problem solving, develop ideas [3], fosters critical and creative nature, thinking satisfaction and pride. A project can contain a number of problems and therefore the completion of the project becomes very important in mathematics. NCTM (National Council of Teachers of Mathematics) states that the "problem solving is very important of mathematics learning” (Babakhani, 2011) and the releasivce of the material studied is assured, but also the experience of learning Becomes more exciting and more meaningful (Barrows and Tamblyn, 1980; quoted from [7].

In solving problems, it is known that students of basic education for all ages and abilities experience difficulties [12]. Difficulties are also faced by students in higher education related to proving theorem [13]. Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies [12]. Therefore, to identify difficulties and provide guidance, the code of thinking theorem is used. Through the code, the lecturer can estimate the level of scaffolding to achieve the suitability of the code thinking problem given [13]. Scaffolding as a form of assistance is intended to access thinking, affecting the potential of students so students can relearn the project and be able to solve it. This study examines, describing the results of students' thinking investigations and rebuilding their thinking structure through scaffolding in completing projects on PBL activities.

2. Research Method

Projects given to students include binomial series theorems in the Maclaurin series. The theorem is a topic in the Calculus course. This course is given to students of informatics engineering major in Bontang High School of Technology. The project in question is the following

Theorem Binomial Series [14]
For any real number p and for |x| < 1,

\[(1 + x)^p = 1 + \binom{p}{1}x + \binom{p}{2}x^2 + \binom{p}{3}x^3 + \ldots\]

question:
(1) Investigate the suitability of form translation \((1+x)^p\) by applying the Maclaurin series formula!
(2) Apply the binomial theorem to 31 tribes to find the closest value of \((1.5)^{1/2}\) and compare the results with the calculator calculation!

This project, we call as "project calculus" and is equipped with a code of thinking of its completion. There are two thinking codes used, namely the thinking code according to Figure 1 for the first question and the thinking code according to Figure 2 for the second question.

![Figure 1. Project of thinking code](image)

The thinking code description in accordance with Figure 1 is presented as table 1.
Table 1. Code of project thinking

<table>
<thead>
<tr>
<th>Thinking Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1, P1</td>
<td>Translate, and select information needed and develop a solution strategy</td>
</tr>
<tr>
<td>Q2</td>
<td>Select the x value</td>
</tr>
<tr>
<td>Q3</td>
<td>Select a function</td>
</tr>
<tr>
<td>Q4 – Q8</td>
<td>Function derivatives up to 4th derivative</td>
</tr>
<tr>
<td>Q9 – Q13</td>
<td>Substitution of x value for each function derivative</td>
</tr>
<tr>
<td>Q14</td>
<td>Presents in maclaurin series</td>
</tr>
<tr>
<td>P2 – P5</td>
<td>Selection of each combination as the coefficient</td>
</tr>
<tr>
<td>P6 – P9</td>
<td>Simplifies combinations</td>
</tr>
<tr>
<td>P10</td>
<td>Presents in unraveled form</td>
</tr>
<tr>
<td>R1</td>
<td>Compare the result of Q14 and P10 to compile the final conclusions</td>
</tr>
</tbody>
</table>

![Image of project of thinking code]

Figure 2. Project of thinking code

The thinking code description in accordance with Figure 2 is presented as in Table 2

Table 2. Code of thinking projects

<table>
<thead>
<tr>
<th>Thinking Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>Identify compatibility with Maclaurin series</td>
</tr>
<tr>
<td>N2</td>
<td>Change the shape of 1.5 in x + 1</td>
</tr>
<tr>
<td>N3</td>
<td>Finds the value of x</td>
</tr>
<tr>
<td>N4</td>
<td>Find p value</td>
</tr>
<tr>
<td>N5</td>
<td>Use information known to the first tribe</td>
</tr>
<tr>
<td>N6 – N10</td>
<td>Substitution of the value of x and p value in combination until the 19th tribe</td>
</tr>
<tr>
<td>N11 - 15</td>
<td>Calculate the combination of p and x substitution using excel</td>
</tr>
<tr>
<td>N16</td>
<td>Find the number of the first 15 tribes</td>
</tr>
<tr>
<td>N17</td>
<td>Calculate with a calculator</td>
</tr>
<tr>
<td>N18</td>
<td>Compare and identify the level of accuracy of calculations</td>
</tr>
</tbody>
</table>

The content contained in the project includes two things, namely (1) the verification of binomial theorems that are solved by implementing Maclaurin series and (2) finding the results of calculations with the approach value. In the calculation used, there are a number of numbers that allow high or low values with sufficient numbers of digits. Therefore, students can use calculating aids and mathematical software. Its use in calculations is recommended because in accordance with the statement that "another trend in mathematics education is the use of mathematical software, which also makes insight more important than technical solving skills" [5]. Besides that, critical thinking is enhanced and students can easily conduct their inquiry as well as innovate by exploiting, sometimes the advantages of technology [10].
The study only involved one group consisting of two students with one project that contained two problems. Project completion activities are carried out outside routine lecture activities. The use of many projects involving larger classes will be an advanced research topic. Project completion planning is arranged according to a mutually agreed schedule, namely:

1. The first day, students receive the project and they seek information, supporting topics related to the project.
2. The second day, the presentation of the results of obtaining supporting information.
3. Third day, presentation of project completion (stage 1)
4. The fourth day, the presentation of project completion (stage 2)

In accordance with the schedule, students receive the project on August 20, 2018 and the completion period ends on August 23, 2018. During the completion of the project by students, researchers conduct observations and interviews. The results of the research activities were used to consider providing scaffolding levels according to the code of thinking. The scaffolding will focus on the second day until the fourth day. Scaffolding levels only describe the second level, namely explaining, reviewing and restructuring and level three, namely developing conceptual thinking [15]. All data collected in the study were analyzed qualitatively which included data reduction, data presentation, conclusions and verification [16].

3. Result And Analysis

The project definition used in this study is a form of investigation that involves a process of transformation and construction of knowledge [11]. Students conduct investigations in group conditions consisting of two students. The goal is to be able to interact with each other, exchange ideas, and develop ideas. During the investigation process, students apply their cognitive abilities and problem solving strategies. The cognitive phase in problem solving includes translating, integrating, planning and implementing [17].

3.1 Investigation on first day

KH and WD's activities during the investigation on the first day showed their success in collecting all information related to the project. The information in question includes combination, Maclaurin series, nth function derivative, terms in series and general sequence. The combination notation shown is \( n \text{Cr} \) or \( \binom{n}{r} \) by definition

\[
n \text{Cr} = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)...(n-r+1)}{r!}
\]

symbol “n!, r!, and (n-r)! is factorial”. The Maclaurin series equation used is

\[
f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + \ldots
\]

notation \( f''(0) \) as the 1st derivative, \( f''(0) \) as 2nd derivative, \( f''(0) \) as the 3rd derivative and the intended term is \( f(0) \) as the 1st tribe, \( \frac{f'(0)}{1!}x \) as the 2nd tribe, \( \frac{f''(0)}{2!}x^2 \) as the 3rd tribe. In accordance with the information that has been given, the two students do not need to be in ZPD to receive scaffolding. This is because information that supports the completion of the project has been fulfilled.

3.2 Investigation on second day

KH and WD presented examples of combinations that use \( \binom{n}{r} \) = \( \frac{n!}{(n-r)!r!} \). Namely \( 5C3 = \frac{5!}{(5-3)!3!} = \frac{5.4.3!}{3.2!} = 10 \). However, the use of notation \( \binom{n}{r} = \frac{n(n-1)(n-2)...(n-r+1)}{r!} \) stating the combination failed to translate in the form \( n(n-1)(n-2)...(n-r+1) \).

The results shown in counting \( \binom{5}{2} \) are \( \frac{5(5-1)(5-2)...(5-2+1)}{2!} = \frac{5.4.3.2.1.4}{2!} = 240 \). Even though the correct result is \( \binom{5}{2} = 60 \). To review the thinking process, students are asked to observe examples and invite them to draw conclusions based on examples. The examples referred to are in accordance with figure 3 given.

![image](image.png)

Figure 3. Example calculation
3.3 Investigation on the third day

Discussion snippet 2

KH & WD: “Can we be given a figure to prove it? We are confused about what needs to be done, sir.”

D: “What information have you obtained for the binomial theorem?”

KH & WD: “Combination, sir.”

D: “What more information?”

KH & WD: “Maclaurin series”

D: “Please describe the combination of the binomial theorem and which one is the function and describe the derivatives of the function. After that, compare the results to get a conclusion.”

KH & WD: “Okay, sir. Thank you for giving his views in proof. We will try it first, sir.”

After discussion, the activities of KH and WD in translating and interpreting the first question are grouping, identifying the most important parts between the binomial theorem and the Maclaurin series. The most important part of the binomial theorem is the Maclaurin combination and series of function derivatives, x values, and the Maclaurin series formula formula. The planning is to trace the values of the terms of the Maclaurin series, simplify the combination form on the binomial theorem, and compare the results of both. The description of the binomial theorem by students can be observed in Figure 4 and the Maclaurin series can be observed in figure 5.

In accordance with Figures 4 and 5, it appears that students describe the form \( \binom{p_1}{1}, \binom{p_2}{2}, \binom{p_3}{3} \) for binomial theorems and \( f'(0), f''(0), f'''(0) \) for the Maclaurin series as coefficients of \( x, x^2, x^3 \) respectively. The team then compares the results of both (combinations and derivative functions) to draw conclusions.

3.4 Investigation on the fourth day

Discussion snippet 3

At first students experience confusion in determining the direction of translation until the calculation. The lecturer then provides his views on the direction of his translation by relating the evidence that the student has done to the first question. It is known (see figures 4 and 5) that students have described \( \binom{p_1}{1}, \binom{p_2}{2}, \binom{p_3}{3} \) or \( f'(0), f''(0), f'''(0) \). Meanwhile, there are 27 unidentified tribes \( \binom{p_1}{1}, \binom{p_2}{2}, \binom{p_3}{3} \) or \( f'(0), f''(0), f'''(0) \) which need to be identified.

[Discussion snippet 4]
KH & WD : "Sir, we have not been able to translate the purpose of this question?"
D : "how many tribes are known? how many tribes are asked? identify the unknown tribes!"

By using the elaboration form as shown in figures 4 and 5, the two students managed to identify the 27 tribes that were unknown with the results according to figure 6.

\[
\begin{align*}
\left( \frac{p}{4} \right) & = \frac{f^{(0)}(0)}{4!} = \frac{p(p-1)(p-2)(p-3)}{4!} \\
\left( \frac{p}{5} \right) & = \frac{f^{(0)}(0)}{5!} = \frac{p(p-1)(p-2)(p-3)(p-4)}{5!} \\
\left( \frac{p}{6} \right) & = \frac{f^{(0)}(0)}{6!} = \frac{p(p-1)(p-2)(p-3)(p-4)(p-5)}{6!} \\
\left( \frac{p}{30} \right) & = \frac{f^{(0)}(0)}{30!} = \frac{p(p-1)(p-2)\ldots(p-29)}{30!}
\end{align*}
\]

Figure 6. Results of tribal identification

[Discussion snippet 4]
D : "How are the identification results"
KH & WD : (showing the results)
D : "Try serving it in full!"

All identified tribes can be written in full as shown in figure 7

\[
(1+x)^{\frac{p}{2}} = 1 + \frac{p(p-1)}{2!}x + \frac{p(p-1)(p-2)}{3!}x^2 + \ldots + \frac{p(p-1)(p-2)\ldots(p-29)}{30!}x^{30}
\]

figure 7. Overall description

until he rewrites the entire tribe in question, students still have difficulties to follow up.

The lecturer again reminded the second question, which is doing calculations. Meanwhile there has been a form (showing the whole tribe) and asked to calculate \(1.5^{1/2}\) using that form. The next student activity is to change \(1.5^{1/2}\) to form \((1 + x)^{p}\) and identify the corresponding \(x\) and \(p\) values. The results of the identification obtained obtained that \(x = 0.5\) and \(p = \frac{1}{2}\).

[Discussion snippet 5]
D : "the binomial theorem covers the form \((1 + x)^{p}\) that you have identified. Meanwhile we are asked to determine the value of \(1.5^{1/2}\) using the theorem. What do you need to do first?"
KH & DW : "still confused, sir"
D : "what variables are there in \((1 + x)^{p}\)? p as what and x as what?"
KH & DW : "x and p, p as rank and x as form \((1 + x)\)"
D : Try to see \(1.5^{1/2}\), what rank and number are raised?
KH & DW : "means, p = \frac{1}{2} and x = 0.5. This blessing, we enter one by one into these 30 tribes, sir?"
D : "yup, cool. because this involves quite large numbers, you can use Excel, Calculator, and Geogebra"

Calculation of the approach value for the results of 1.51/2 using the form

\[
(1 + x)^{p} = 1 + \frac{f^{(1)}(0)}{1!}x + \frac{f^{(1)}(0)}{2!}x^2 + \ldots + \frac{f^{(30)}(0)}{30!}x^{30}
\]

\[
= 1 + px + \frac{p(p-1)}{2!}x^2 + \ldots + \frac{p(p-1)(p-2)\ldots(p-29)}{30!}x^{30}
\]

The calculation process performed by KH and WD involves a combination of excel, calculator, and Geogebra (how close is the calculation between the two). In excel calculations, KH and WD separate for factorial values, function / combination derivative values, and x positions.

From these results, then the factorial value is as a divider for multiplication of the value of the function / combination derivative and x.

a. Factorial results
In accordance with the elaboration of the tribes that KH and WD have done, they identify values of 1!, 2!, 3!, up to 30! in excel calculations and calculators. Factorial results from 1 to 30 are shown in Figure 8.
In accordance with figure 8, there are differences in the results of calculations between excel and calculators, especially at $25! \leq n \leq 30!$, $n \in \mathbb{Z}$. The difference in calculation results is presented in table 3.

Table 3. Calculations $n!$ Use calculator $n!$ for $25 \leq n \leq 30$, $n \in \mathbb{Z}$

<table>
<thead>
<tr>
<th>$n$</th>
<th>Hasil $n!$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>51,090,342,711,709,440,000</td>
</tr>
<tr>
<td>22</td>
<td>1,124,000,727,777,607,680,000</td>
</tr>
<tr>
<td>23</td>
<td>25,852,016,378,864,976,640,000</td>
</tr>
<tr>
<td>24</td>
<td>620,448,010,733,239,439,360,000</td>
</tr>
<tr>
<td>25</td>
<td>15,511,210,043,330,985,984,000,000</td>
</tr>
<tr>
<td>26</td>
<td>403,291,461,126,605,635,584,000,000</td>
</tr>
<tr>
<td>27</td>
<td>10,888,696,450,418,352,160,766,000,000</td>
</tr>
<tr>
<td>28</td>
<td>304,888,344,611,713,860,504,000,000,000</td>
</tr>
<tr>
<td>29</td>
<td>8,841,761,993,739,701,954,533,616,000,000,000</td>
</tr>
<tr>
<td>30</td>
<td>265,252,859,812,191,058,636,308,480,000,000,000</td>
</tr>
</tbody>
</table>

b. Appointment value $x$

In order to obtain 31 terms in the Maclaurin series, KH dan WD estimate the $x$ rotation is at 1, 2, 3, ..., 30 and for $x$ itself is $x = 0.5$ (obtained by translation $(1 + 0.5)^{1/2} = (1 + x)^{p}$. The calculation results of the $x^n; x = 0.5; n = 1,2,3,...,30$ are presented in Figure 9.

Figure 9. Value $x^n; x = 0.5$ and $n = 1,2,3,...,30$

the findings of calculation of the ranks with different results occur also for the form $0.5^n; 24 \leq n \leq 30, n \in \mathbb{Z}$ are presented in the table.

Table 4. The results of calculating the calculator for $0.5^n; 24 \leq n \leq 30, n \in \mathbb{Z}$

<table>
<thead>
<tr>
<th>$N$</th>
<th>Hasil $0.5^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0,0000000059606444775390625</td>
</tr>
<tr>
<td>25</td>
<td>0,00000000298023223876953125</td>
</tr>
<tr>
<td>26</td>
<td>0,00000000149016119384876525</td>
</tr>
<tr>
<td>27</td>
<td>0,0000000007450580596923828125</td>
</tr>
<tr>
<td>28</td>
<td>0,00000000037252902984619140625</td>
</tr>
<tr>
<td>29</td>
<td>0,000000000186264514923095703125</td>
</tr>
<tr>
<td>30</td>
<td>0,0000000000931322574615478515625</td>
</tr>
</tbody>
</table>

c. The derivative value of the function $f(x) = (1 + x)^p$; according to the definition of Maclaurin series $x = 0$.

Identification of derivatives in the Maclaurin series to obtain 31 tribes, the derivatives involved until reaching the 30th derivative.
Value for \( f'(0) = p(p-1)(p-2)...(p-(n-1)); n = 1,2,3,...30 \) can be observed in figure 10.

![Figure 10. Value of \( f'(0) \) to all 30 tribes](image)

Difference calculation for \( f'(0) = p(p-1)(p-2)...(p-(n-1)); n = 1,2,3,...30 \) occurs at the 11,12,...,30 derivatives. The differences in question can be observed in table 5.

**Table 5. The calculation results of calculators for \( f'(0) = p(p-1)(p-2)...(p-(n-1)); n = 11,...,30 \)**

<table>
<thead>
<tr>
<th>Turunan ke-n</th>
<th>( f'(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>319691,93115234375</td>
</tr>
<tr>
<td>12</td>
<td>335676,277099609375</td>
</tr>
<tr>
<td>13</td>
<td>386028,06866455078125</td>
</tr>
<tr>
<td>14</td>
<td>48255068,58306884765625</td>
</tr>
<tr>
<td>15</td>
<td>65142216,587142943359375</td>
</tr>
<tr>
<td>16</td>
<td>94456279,9301357269287109375</td>
</tr>
<tr>
<td>17</td>
<td>1464071532917,10376739501953125</td>
</tr>
<tr>
<td>18</td>
<td>24157180293132,21216208729225625</td>
</tr>
<tr>
<td>19</td>
<td>422750655129813,721283531180964844</td>
</tr>
<tr>
<td>20</td>
<td>7820887119901553,6874532699584961</td>
</tr>
<tr>
<td>21</td>
<td>152507298830580296,90533876419067</td>
</tr>
<tr>
<td>22</td>
<td>312639962180646086,55944446659088</td>
</tr>
<tr>
<td>23</td>
<td>6727759162883890861,028060317039</td>
</tr>
<tr>
<td>24</td>
<td>1512395819164887544373,1313571334</td>
</tr>
<tr>
<td>25</td>
<td>35541301750374857292768,586992635</td>
</tr>
<tr>
<td>26</td>
<td>870761894284416003672830,37806955</td>
</tr>
<tr>
<td>27</td>
<td>22204428268546692093657174,661173</td>
</tr>
<tr>
<td>28</td>
<td>588417349116487340481915128,5211</td>
</tr>
<tr>
<td>29</td>
<td>1618147710070340186325666034,33</td>
</tr>
<tr>
<td>30</td>
<td>46117209737004695310270091978,41</td>
</tr>
</tbody>
</table>

**[Discussion snippet 6]**

D : "What are the results of calculations that you can find?"

KH & DW : "We find 3 things, sir."

D : "What is that?"

KH & DW : (shows the results of factorial calculations, derivative values, and x positions) "We find a different calculation, sir."

D : "Just continue. Analyze all the differences that you find yes."

KH & DW : "OK, sir"

d. Calculation of the value 0.512 through the Maclaurin series approach.

The calculation result of a value of 0.512 using the Maclaurin series approach involves factorial results data (figure 8), the value of the x position (figure 9), and the derivative value (figure 10). The principle carried out is to add the multiplication results between the data of figure 9 and figure 10 which are then divided by the data of figure 8. The results of the calculations are presented in Figure 11.
In accordance with Figure 11, it is known that the calculation results of up to 31 terms show 1,22474487139.77 trillion. Meanwhile, the direct calculation is 1,22474487139159 trillion. From both calculations, the difference is 0.000000000005218776986899 or 0.5 x 10^{-12}. Whereas for calculating calculators obtained results data as in figure 12.

Based on Figure 12, the number of 30 tribes shows the results of 1,2247448713910671935953618485813. Therefore there is a difference that is 5.218555032801887716456959371498e-13 or 0.5218555032801887716456959371498 x 10^{-12}. If we compare the results of the three calculations, namely excel and calculator, the difference is obtained which reaches 0.000000000000027272631128977164. In the presentation of normal size, the value coincides using Geogebra simulation. The proximity conditions can be observed as shown in Figure 12.
4. Conclusion

This calculus project refers to [11] as an example of a project in PBL that focuses on forms of investigation that involve transformation and construction of knowledge. During the completion of the calculus project, students were very active in investigating and establishing openness during their investigations. Initial investigations are intended to collect all information that supports the completion of the project. This is important because it becomes a guideline for students to follow the advanced completion stage. In principle, failure occurs because of unpreparedness of information that supports activities. The presentation of information resulting from the investigation is part of the form of construction and this can be used as a determinant to identify thinking errors in the project. This study shows that students succeeded in carrying out their investigations in completing the calculus project which was supported by the provision of scaffolding. This can be evidenced from all supporting information on the completion of the project and the presentation of information on the first and second days. Meanwhile, on the third and fourth days, students can compare between their tribes and are able to identify the results of calculations using both calculating aids and mathematical software.

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